Benha University Faculty of Engineering – Shoubra Department of Industrial Engineering Course: Mathematics 1 Code: EMP 101		Final Exam Date: May 20, 2016 Duration: 3 hours Answer **All** questions
• The exam consists of one page		• No. of questions: 4 Total Mark: 40

Question 1

(a)Find y\` from the following: **6**

(i) $y = 2x^4 - 3^x + 3x$ (ii) $y = e^x + x^{-3}$ (iii) $y = \ln x + \ln(x+3)$

(iv) $y = \frac{1}{2} + \frac{\cos x}{x^3}$ (v) $y = \cos x \, . \tan x$ (vi) $y = \sin^4 x + \sin x$

(b) Determine the maximum and minimum points of : $f(x) = 4x^3 - 12x + 1$ **3**

(c) Write the Maclurin's expansion of : $f(x) = x.e^x$ **2**

Question 2

Find the integrals: **9**

(a) $\int (x^2 - 2^x + 2x)\, dx$ (b) $\int (1 - x^2)^2\, dx$ (c) $\int (3 + x + e^x)\, dx$

(d) $\int (3^x - 2^x)^2\, dx$ (e) $\int (\sqrt{x} + \frac{1}{x^3})\, dx$ (f) $\int (3\cos x + \sin 3x)\, dx$

Question 3

(a) If $A = \begin{bmatrix} 1 & 3 & 0 \\ 4 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ **7**

Find, if possible, A + B, A + C, A.B, $A.B^t$, C.B, |A| and |C|

(b) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$. **4**

Question 4

(a) Solve the linear system : x + 2y + z = 2, – x + y + 3z = 0, 3y + 4z = 5. **3**

(b)Using the binomial theorem, expand $\frac{1}{\sqrt{1-2x}}$ **3**

(c)If $z_1 = 3 - 2i$, $z_2 = -1 + 3i$. Find $z_1 + z_2$, $z_1 . z_2$, z_1 / z_2 . **3**

Good Luck *Dr. Mohamed Eid*

Model Answer

Question 1

(a)(i) $y` = 8x^3 - 3^x . \ln 3 + 3$

(ii) $y` = e^x - 3x^{-4}$

(iii) $y` = \frac{1}{x} + \frac{1}{x+3} \ln x$

(iv) $y` = 0 + \cos x . (-3x^{-4}) - \sin x . x^{-3}$

(v) $y` = \cos x . \sec^2 x - \sin x . \tan x$

(vi) $y` = 4\sin^3 x . \cos x + \cos x$

--*6 Marks*

(b) Since $f`(x) = 12x^2 - 12 = 0$. Then x = 1, x = – 1.

From $f``(x) = 24x$. We see that $f``(1) = 24$, then x = 1 is minimum.

We see that $f``(-1) = -24$, then x = –1 is maximum.

--*3 Marks*

(c) Since $f`(x) = x.e^x + e^x$, then $f`(0) = 1$

$f``(x) = x.e^x + 2e^x$, then $f``(0) = 2$

$f``(x) = x.e^x + 3e^x$, then $f``(0) = 3$

Then $f(x) = 1 + 2x + \frac{3}{2!}x^2 + \cdots$

--*2 Marks*

Question 2

(a) $\int (x^2 - 2^x + 2x)\, dx = \frac{1}{3}x^3 - \frac{1}{\ln 2}2^x + x^2 + c$

(b) $\int (1 - x^2)^2\, dx = \int (1 - 2x^2 + x^4)\, dx = x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + c$

(c) $\int (3 + x + e^x)\, dx = 3x + \frac{1}{2}x + e^x + c$

(d) $\int (3^x - 2^x)^2\, dx = \int (9^x + 4^x - 2.6^x)\, dx = \frac{1}{\ln 9}9^x + \frac{1}{\ln 4}4^x - \frac{2}{\ln 6}6^x + c$

(e) $\int (\sqrt{x} + \frac{1}{x^3})\, dx = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{-2} + c$

(f) $\int (3\cos x + \sin 3x)\, dx = 3\sin x - \frac{1}{3}\cos x + c$

--*9 Marks*

Question 3

(a) A + C, A.B, |A| are impossible.

$A + B = \begin{bmatrix} 4 & 3 & 1 \\ 6 & -1 & 4 \end{bmatrix}$ and $A.B` = \begin{bmatrix} 1 & 3 & 0 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 13 & 9 \end{bmatrix}$

$C.B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 5 \\ 18 & 3 & 13 \end{bmatrix}$

|C| = 6 – 4 = 2

---*7 Marks*

(b) $\begin{vmatrix} 0-\lambda & 3 \\ 1 & 2-\lambda \end{vmatrix} = -\lambda(2-\lambda) - 3 = \lambda^2 - 2\lambda - 3 = 0$. Then $\lambda_1 = -1, \quad \lambda_2 = 3$

From the equation, $\begin{bmatrix} -\lambda & 3 \\ 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = -1$, $\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then   x + 3y = 0,   x + 3y = 0

Then   x = – 3y = any number except 0

Put y = 1, we get x = – 3 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

For : $\lambda_2 = 3$, $\begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then –3x + 3y = 0,   x – y = 0

Then   x = y = any number except 0

Put y = 1, we get x = 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

---*4 Marks*

Question 4

(a) $G = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 2 \\ -1 & 1 & 3 & 0 \\ 0 & 3 & 4 & 5 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 2 \\ 0 & 3 & 4 & 2 \\ 0 & 3 & 4 & 5 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 2 \\ 0 & 3 & 4 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right]$

We see that rank A is 2 but rank G is 3. Then, there is no solution.

---*3 Marks*

(b) $\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1 + x + \frac{3}{2}x^2 + \cdots, \quad |2x| < 1$

---*3 Marks*

(c) $z_1 + z_2 = 2 + i, \quad z_1.z_2 = 3 + 11i, \ \frac{z_1}{z_2} = -\frac{9}{10} - \frac{7}{10}i$

---*3 Marks*

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